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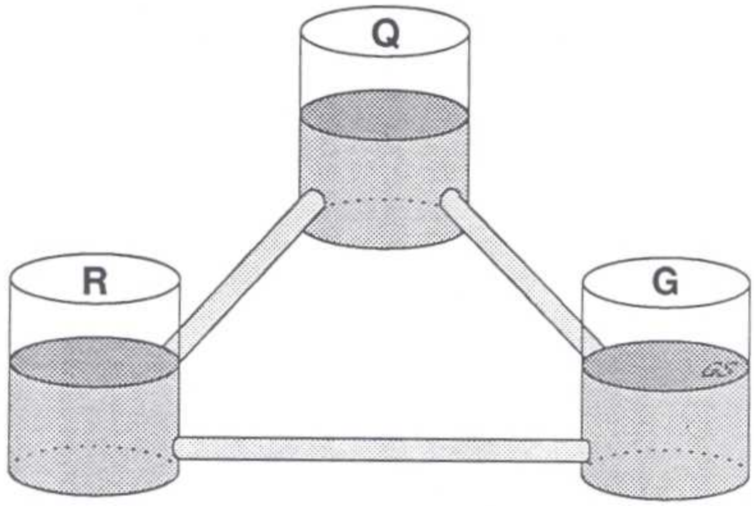

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FINDING STABLE CAUSAL INTERPRETATIONS OF EQUATIONS

by

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ABSTRACT

The *causal ordering* procedure of Iwasaki and Simon [5,6] provides a means for uncovering causal dependencies among variables constrained by a set of mathematical equations. This paper examines the procedure from a qualitative modeling viewpoint and addresses one of its limitations: *context sensitivity.* Causal dependencies predicted by the procedure may change depending on the context or scenario in which the underlying physical system operates. This prevents the qualitative modeler from using causal ordering to determine, a priori, a single causal interpretation of equations describing some phenomenon. We show that in some cases it is possible to find clusters of equations that possess *causal stability.* That is, their causal dependencies are the same in all scenarios consistent with the equations' *modeling viewpoint.* These *unidirectional* equations help the qualitative modeler by providing a stable, unambiguous causal interpretation. To identify such equations we define conditions sufficient to guarantee causal stability. In addition, we show that unidirectional equation sets are *causally independent* of equations outside their set. Thus, they add compositionality to the causal modeling task. Lastly, we demonstrate our ideas by uncovering the causal dependencies of Hooke's law, Gauss's law for electricity, and Bernoulli's equation.

1 INTRODUCTION

The notion of cause and effect plays a major role in human commonsense reasoning. Since causation explains how systems achieve their behavior, it has been widely adopted as a means of understanding the events of everyday life. In science and philosophy however, causation remains a controversial topic. At one extreme, proponents of the *causality doctrine* claim that all connections between events in the world are ultimately causal. In other words, nature can be described as a complex graph of cause and effect links [1]. Others view causation as a myth, occuring mostly in the primitive stages of a science. Bertrand Russell declared that "...in a sufficiently advanced science, the word 'cause' will not occur in any statement of invariable laws" [7]. Whether or not nature possesses true cause and effect relationships, these notions pervade our explanations of natural phenomena. Since a major goal of *qualitative physics* [8] is computer modeling of commonsense reasoning about natural phenomena, it should include some notion of cause.

There are widely varying accounts of causation among different theories of qualitative physics. A key distinction among them is whether or not connections between quantities are *directed*. In de Kleer and Brown's *confluence* theory [2], equations represent undirected constraints between quantities. Perturbations among any $n-1$ of the quantities in an equation cause the nth quantity to change. de Kleer and Brown's approach is not causal in the classical sense since it lacks the essential ingredient of *unidirectionality*. In their theory, causation can flow through an equation in many ways. In contrast, Forbus' *qualitative process* (QP) theory [3] asserts that connections between quantities have a fixed causal direction. Quantities interact with each other through *direct* and *indirect* influences. A quantity is directly influenced if and only if it is acted upon by a *process*. Examples of processes are motion, acceleration, boiling and heat flow. When a motion process acts on an object, it directly influences its position and causes that position to change over time. A quantity is indirectly influenced when it is a function of another quantity that is changing. For example, when a force f acts on a particle of mass m, the particle's acceleration is indirectly influenced by the applied force. Changes in applied force instantly cause acceleration to change, but not vice versa. In the language of QP theory, acceleration is qualitatively proportional to applied force.

Unfortunately, the directed connectives of QP theory make it difficult for the qualitative modeler to represent certain laws of nature. The model builder is forced to decide a priori which way causation flows in all circumstances. But many laws take the form of equations which have no obvious causal interpretation. Consider for example Bernoulli's equation for liquid flow along a fixed path: $P+\frac{1}{2}\rho v+\rho g y=c$, where P is pressure, ρ is fluid density, v is velocity, g is the gravitational acceleration constant, y is height, and c is a constant. Is pressure causally dependent on velocity, or vice versa? How does the modeler uncover asymmetries in the interactions, and are the asymmetries valid in all circumstances? The *causal ordering* procedure presented by Iwasaki and Simon [5,6] addresses part of this

problem by attempting to extract causation directly from mathematical equations. As we shall see though, the procedure is *context sensitive.* The predicted causal dependencies of an equation may change depending on the context or scenario in which the underlying physical system operates. In other words, the model builder cannot be certain that the causal ordering of a system's equations remains unchanged when that system is placed in different environments. However, we will show that in certain cases, it is possible to find clusters of unidirectional equations whose causal ordering is stable across all scenarios consistent with the equations' *modeling viewpoint.* Unidirectional equations ease the modeler's burden in two ways. First, they have an unambiguous causal interpretation. Second, as we will show, they add compositionality to the difficult task of causal modeling.

2 CAUSAL ORDERING

The causal ordering procedure begins with a mathematical model of a phenomenon in the form of n independent equations, $\vec{f}(\vec{x}) = \vec{c}$, in n unknowns, $\vec{x} = \{x_1, \ldots, x_n\}$. In order for the procedure to yield causal relations that are intuitively plausible, each equation should be *structural.* That is, each equation should reflect a mechanism operating at some level of detail in the phenomenon. Unfortunately, there is no simple way to know if an equation is structural. One useful heuristic is that a structural equation's variables should represent things which interact locally through processes. For example, Hooke's law, $f = -kx$, relating the internal force f of a spring and its displacement x, is structural. The equation is a rough reflection of the interactions occuring between the spring's atomic bond forces and deformation of its crystalline structure.

We illustrate the causal ordering procedure by applying it to the bathtub example shown in Figure 1a (adapted from [6]). Water enters the bathtub at a mass flow rate of $m_{r,i}$ and leaves at a rate of $m_{r,o}$. P designates water pressure at the bottom of the bathtub and M is the total mass of water in the bathtub. Assuming steady state behavior[1], the system is modeled by the equations of Figure 1b. The equations have the following interpretation (the c_i's are constants): (1) $P = c_1 M$, pressure is proportional to the mass of water in the tub; (2) $m_{r,o} = c_2 P$, the rate of flow out of the tub is proportional to pressure; (3) $m_{r,o} = m_{r,i}$, the rates of flow into and out of the tub are equal at steady state; (4) $m_{r,i} = c_4$, the input rate of flow is *exogenous.* That is, the input rate is controlled by factors outside the bathtub system.

The procedure assigns causal dependencies between variables by propagation through a structural equation matrix (Figure 1c). Each matrix element is either zero (blank) or marked with a "1". Each row has one or more marks. A mark in row i column j means that variable x_j appears in equation i. In order for the procedure to work, the matrix must be expressible in upper triangular form with 1's along the diagonal. The matrix fails

[1] A system operates at steady state when its properties at any point in space are constant over time.

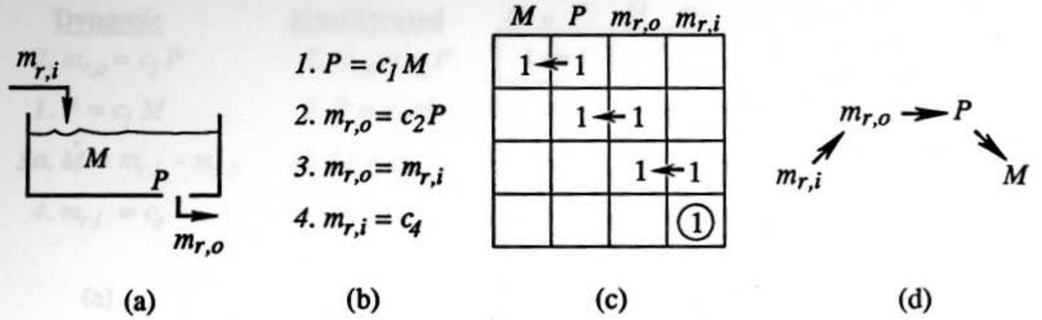


Figure 1: Steady State Bathtub Example

to meet this condition when the equations are either underconstrained or overconstrained. For example, in phenomena containing feedback, the structural equation matrix cannot be expressed in upper triangular form—one or more terms below the diagonal is marked and the equations are underconstrained. If the equations are overconstrained, some diagonals are zero and the causal dependencies may be inconsistent.

By definition, one or more rows of an upper triangular matrix must contain a single mark. A row containing a single mark in column j represents an exogenous variable x_j. Exogenous variables are the causal inputs to the phenomenon and are unaffected or negligibly affected by it. It is from these variables that causation propagates to other variables. In this paper, exogenous variables such as $m_{r,i}$ in Figure 1c, are circled in their structural equation matrices. The final causal structure or *influence* diagram is shown in Figure 1d. The diagram shows that M is causally dependent on P, which depends on $m_{r,o}$, which in turn depends on $m_{r,i}$. If the input flow rate is increased, the output flow rate must increase after reaching steady state. In this steady state model, pressure can be viewed as a measure of the bathtub's resistance to flow. If the flow rate of water through the bathtub is increased, pressure increases. Less intuitively, if the pressure is increased, mass must accumulate[2].

In [6], Iwasaki extends the procedure to handle first order, ordinary differential equations. The procedure requires that differential equations be translated into "canonical form". Each differential equation must contain only one time derivative, as in $\dot{x}_i = f_i(\vec{x})$, and each derivative must appear in only one equation. A variable whose derivative appears in an equation is said to be causally dependent on the other variables in the equation. For those variables which do not appear as derivatives in any equations, causal dependencies are found, as before, by propagation through a structural equation matrix. The matrix is

[2] This causal dependency seems unintuitive because bathtubs are not ordinarily viewed as steady state systems. If we study analogous phenomena which *are* usually viewed as steady state, their causal orderings seem more natural. Consider the phenomenon of steady current flow through a resistor in parallel with a capacitor. Current flow i corresponds to the two bathtub flow rates. Voltage drop and capacitor charge accumulation correspond to pressure and water mass, respectively. As more voltage is applied to the circuit, charge accumulates on the capacitor plate.

Dynamic

2. $m_{r,o} = c_2 P$

1. $P = c_1 M$

3a. $\dot{M} = m_{r,i} - m_{r,o}$

4. $m_{r,i} = c_4$

(a)

Equilibrated

2. $m_{r,o} = c_2 P$

1. $P = c_1 M$

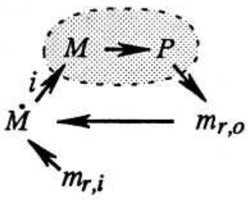
3b. $M = c_3$

4. $m_{r,i} = c_4$

(b)

$m_{r,o}$	P	M	$m_{r,i}$
1 ←	1		
	1 ←	1	
		1	
			1

(c)



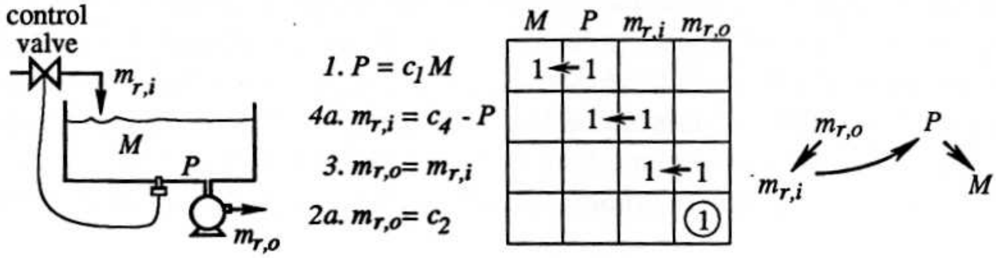
(d)

Figure 2: Dynamic Bathtub Model

formed by "equilibrating" the differential equations, that is, by replacing each differential equation $\dot{x}_i = f_i(\vec{x})$ with the constant equation $x_i = c_i$. These newly introduced constants act as "seeds" or starting points for causal propagation to other variables.

An example will help clarify the extended procedure. Consider again the bathtub system, but viewed as a dynamic system with transient variations in the amount of water (Figure 2). Equation (3a) models the fact that the rate of accumulation of water $\dot{M}$ is equal to the net flow rate of water into the bathtub. Thus, the mass of water is said to be causally dependent on the flow rates into and out of the bathtub, and this is reflected in the causal structure diagram of Figure 2d. The edges marked with an i are causal integration links between a derivative of a variable and the variable itself. In QP theory terms, the variable M is *directly influenced* by the mass flow processes.

Figure 2b shows the equilibrated version of the equations. Equation (3a): $\dot{M} = m_{r,i} - m_{r,o}$, is replaced by the constant equation (3b): $M = c_3$. The intuition behind constant equation (3b) is that the directly influenced quantity M changes so slowly in comparison to other variables that it can be regarded as constant. M is influenced by an integration link and integration requires finite time to cause a finite change. In contrast, the algebraic equations represent mechanisms that are instantaneous in comparison. Figure 2c shows the resulting structural equation matrix. Notice that the differential equation provided a seed, M, at which propagation through the matrix could begin. This "quasi-exogenous" variable is circled with a dotted line in Figure 2c. Notice also that exogenous variable $m_{r,i}$ was not needed for causal propagation through the equilibrated equations.



(a) (b) (c) (d)

Figure 3: New Steady State Scenario Changes Causal Dependencies

3 CAUSAL STABILITY

In general, the causal ordering procedure requires knowing which variables are exogenous. That is, it requires knowing which variables are affected by factors outside of the system being modeled. Since placing a system in different situations changes the interactions between it and its surroundings, the set of variables that are exogenous may change from situation to situation. As a result, the causal ordering procedure is unstable or context sensitive. Causal dependencies uncovered by the procedure for a given equation may change when the equation is applied to a slightly different scenario. In this section we illustrate causal instability with an example and show how, in some cases, it can be avoided.

Consider the bathtub system operating at steady state. We will show that equation (3): $m_{r,o} = m_{r,i}$, is context sensitive. Its causal dependency flips depending on the circumstances. In the simple scenario of Figure 1a, the output flow rate $m_{r,o}$ was shown to be causally dependent on the input flow rate $m_{r,i}$. Next consider the steady state bathtub in the new situation of Figure 3a. The bathtub drain has been attached to a pump which is externally controlled. The output flow rate $m_{r,o}$ has become exogenous. The input stream has been attached to a control valve which varies the input flow rate in an attempt to maintain constant pressure and hence a constant amount of water in the bathtub. The input flow rate is thus no longer independent of the system and cannot be treated as exogeneous. Equations (1) and (3) of the original, simple situation (Figure 1b) also apply in this new scenario. However, because the exogenous variables have changed, causal ordering yields a new dependency for equation (3). Compare the causal influence diagrams of Figures 1d and 3d. In the new situation, the causal link between input and output flow rates has been reversed. Input flow rate $m_{r,i}$ is now causally dependent on $m_{r,o}$. Unfortunately for the qualitative modeler, the direction of causal flow in equation (3): $m_{r,o} = m_{r,i}$, can't be known beforehand.

The causal instability of a set of equations is best understood in terms of its underlying *modeling viewpoint* and *scenario space*. A modeling viewpoint is the set of decisions made by the modeler when conceptualizing the phenomenon. This includes decisions about on-

tology (eg., what individuals and processes exist), spatio-temporal grain size, idealizations, perspective, and operating assumptions. A modeling viewpoint is a necessary prerequisite for formulating a mathematical model. In the steady state bathtub examples, the modeling viewpoint includes the ontological choice of a contained liquid and the idealization of steady state behavior. The dynamic bathtub modeling viewpoint extends the recognized behaviors to include transient variations in the amount of water.

An equation set's scenario space is the set of possible situations that are consistent with the equations, subject to the underlying modeling viewpoint. For example, the scenario space of equations (1): $P = c_1 M$, and (3): $m_{r,o} = m_{r,i}$, includes both situations shown in Figures 1a and 3a. It does not include situations where, for example, boiling occurs in the bathtub, since equation (3) would then imply that the mass of water M would decrease, contradicting the assumed steady state behavior.

We have seen that equation (3)'s causal ordering varies depending on its situational context. In other words, its causal ordering is unstable over its scenario space. This raises the question of whether equations ever possess *causal stability*:

Definition 1 (Causal Stability) A set of algebraic equations at a particular modeling viewpoint is *causally stable* if and only if its causal ordering is invariant with respect to its scenario space. Such a set of equations is unidirectional with respect to the modeling viewpoint.

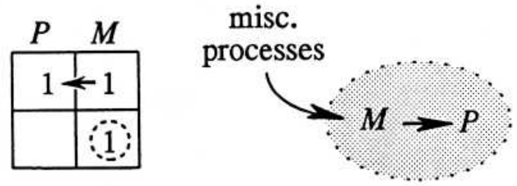
The usefulness of this property is that it allows the qualitative modeler to assign an a priori causal direction to algebraic equations without regard to the particulars of the situations modeled. In QP theory, causally stable algebraic equations can be represented by unique qualitative proportionalities which hold in any scenario that might be encountered.

To demonstrate causal stability, consider again the dynamic bathtub system. We argue that equation (1): $P = c_1 M$, is unidirectional with respect to its underlying modeling viewpoint. In this dynamic viewpoint we recognize that the bathtub may experience transient variations in the mass of water. In QP theory terms, we have acknowledged the possibility of assorted processes which directly influence mass M. This implies that mass M is either constant[3] or quasi-exogenous. In other words, in all dynamic bathtub scenarios M is a seed from which causation propagates to other variables. As shown in Figure 4b, causation propagates from M to P in equation (1). Pressure P is causally dependent on mass M, and this dependency must hold over equation (1)'s entire scenario space. If equation (1)'s causal dependency ever changed, M would be an effect instead of a causal input, thus contradicting the underlying modeling viewpoint. Therefore, equation (1) is causally stable or unidirectional. This stable relation is shaded in the influence diagrams of Figures 2d and 4c.

[3] Eg., the bathtub may be sealed shut.

$$1.\ P = c_1 M$$
$$3b.\ M = c_3$$

(a)



(b) (c)

Figure 4: Causal Stability in the Dynamic Bathtub System

Equation (1)'s scenario space is large. It includes the simple situation of Figure 1a, the process control situation of Figure 3a, situations where boiling occurs, and many others. It does not include situations where, for example, the shape of the bathtub is changed[4] or the bathtub is exposed to vacuum[5]. As long as we remain at the same modeling viewpoint, pressure is causally dependent on mass regardless of the vagaries of the particular scenario. This satisfies an intuitive sense that there is some basic mechanism operating between the amount of liquid and the pressure at the bathtub bottom. The mechanism should have a causal interpretation that is independent of whether the bathtub is attached to a pump, boiling occurs, and so forth.

This example illustrates how the following causal *stability conditions* can be used to find unidirectional equations:

Lemma 1 (Stability Conditions) If $\mathcal{E}$ is a set of algebraic equations satisfying the two conditions: *(i)* the subset of exogenous, quasi-exogenous, or constant variables in $\mathcal{E}$ is unchanged over its scenario space, and *(ii)* the structural equation matrix of $\mathcal{E}$ (including its constant equations) is expressible in $n \times n$ upper triangular form, then $\mathcal{E}$ is unidirectional with respect to its modeling viewpoint.

This follows almost immediately from the definition of causal stability. Over $\mathcal{E}$'s scenario space, the structural equation matrix for the n equations is unchanged, thus guaranteeing invariant causal dependencies.

When causally stable equations exist, they add compositionality to the causal modeling task. In a complex scenario with many equations, unidirectional subsets have causal interpretations that are independent of equations outside the subset. Causally stable equations can be studied in isolation and used as building blocks in many scenarios. In the dynamic bathtub example, equations (1) and (3b) are *causally independent* of other algebraic equations that may arise in various scenarios.

[4]This could change pressure P without changing M, contradicting equation (1).

[5]In a vacuum, liquids vaporize, contradicting the assumed existence of a contained liquid entity.

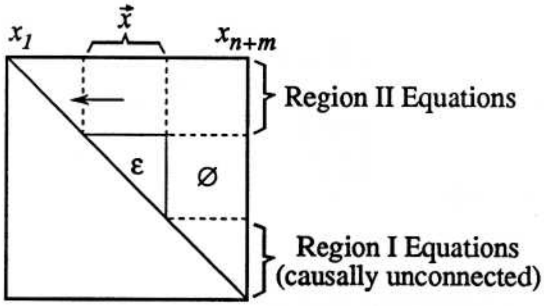


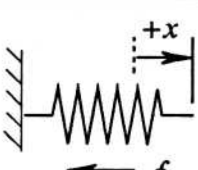
Figure 5: Structural Equation Matrix for $\mathcal{E} \cup \mathcal{E}'$

Theorem 1 (Causal Independence) Let $\mathcal{E}$ be a set of n unidirectional equations in variables $\vec{x}$. Let $\mathcal{E}'$ be a set of m equations, independent of $\mathcal{E}$ and at the same modeling viewpoint. If $\mathcal{E} \cup \mathcal{E}'$'s structural equation matrix is expressible in upper triangular form, then the causal dependencies within $\vec{x}$ are the same in both $\mathcal{E}$ and $\mathcal{E} \cup \mathcal{E}'$.

To prove this, consider $\mathcal{E} \cup \mathcal{E}'$'s structural equation matrix. Without loss of generality, the unidirectional equations $\mathcal{E}$ can be grouped together in the matrix as shown in Figure 5. The added equations $\mathcal{E}'$ must appear in matrix Regions I or II, or both. Equations in Region I share none of the variables of $\mathcal{E}$. Thus $\mathcal{E}$'s variables, $\vec{x}$, are causally unaffected by variables in Region I. Before considering Region II, note that for each row i, the diagonal variable x_i is either a causal seed or is causally dependent on variables $x_j, j > i$. Equations in Region II that share variables with $\mathcal{E}$ provide a path for causal propagation *from* $\vec{x}$ to other variables lying along Region II's diagonal. Thus $\mathcal{E}$'s variables may causally affect variables in Region II, but not vice versa. Therefore, $\mathcal{E}$ is causally independent of $\mathcal{E}'$.

4 OTHER EXAMPLES

Unidirectional equations can be found in many branches of science. Three examples from different domains are shown in Figures 6 through 8. The first example illustrates the stable causal ordering of Hooke's law: $f = -kx$, where f is the force exerted *by* the spring, x



5. $f = -kx$
6. $x = c_6$

	f	x
	1	1
		1

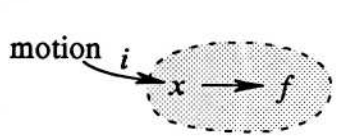
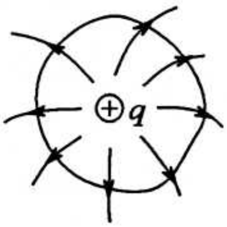


Figure 6: Causal Direction of Hooke's Law



7. $\varepsilon_0 \Phi_E = q$
8. $q = c_8$

$$\begin{array}{cc} \Phi_E & q \\ \hline 1 & \leftarrow 1 \\ & 1 \end{array}$$

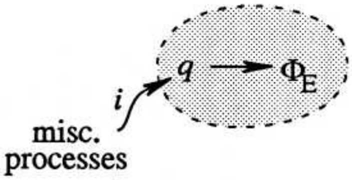


Figure 7: Causal Direction of Gauss' Law

is the displacement from rest length, and k is the spring constant. Our chosen modeling viewpoint is standard in elementary physics. The spring is a homogenous object, subject to smooth motion processes (expansion and compression)[6], and is never stretched beyond its elastic range. Over the space of scenarios consistent with this modeling viewpoint, the displacement x is either constant[7] or directly influenced by motion processes. In other words, x is always constant or quasi-exogenous. Hooke's law plus a constant equation for x form a 2×2 upper triangular structural equation matrix (Figure 6)[8]. Thus, by Lemma 1, Hooke's law is unidirectional with respect to this modeling viewpoint. The spring force f is causally dependent on displacement x in every consistent scenario.

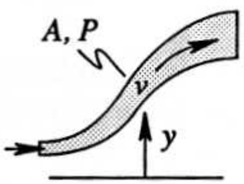
Figure 7 shows the stable causal ordering of Gauss's law for electricity: $\epsilon_0 \Phi_E = q$, where Φ_E is the flux of the electric field through an arbitrary Gaussian surface enclosing an object having net charge q and ϵ_0 is the *permittivity* constant. Of the equations considered so far, this one is unique in its near universal applicability. Every macroscopic object appears to follow Gauss's law for electricity. It is assumed that the mechanisms relating Φ_E and q act instantly in comparison to the rates at which these variables are affected by outside influences[9]. Over the set of possible scenarios, the charge q may be constant or may be influenced by processes. For example, the charge on a capacitor's plate may be influenced by a flow of electrons. In other words, q is always either constant or quasi-exogenous. Gauss's law plus a constant equation for q form a 2×2 upper triangular structural equation matrix (Figure 7). Thus, by Lemma 1, Gauss's law is unidirectional. Φ_E is causally dependent on q. One way of interpreting this is that theories of electromagnetism offer no means of influencing Φ_E except through the intermediary q.

[6] This implies that moving springs either have mass or are always connected to a mass. Otherwise, a spring could equilibrate instantly with an applied external force.

[7] For example, the spring may be held between two immovable objects.

[8] We can safely ignore k's causal effects since it is always constant in this model.

[9] If this assumption was wrong, then the electric flux through two Gaussian surfaces that enclose an object (of increasing charge) could be different depending on how close each surface was to the object.



9. $P + \frac{1}{2}\rho v^2 + \rho g y = c_9$
10. $\rho A v = c_{10}$
11. $A = f_1(x)$
12. $y = f_2(x)$
13. $x = c_{13}$

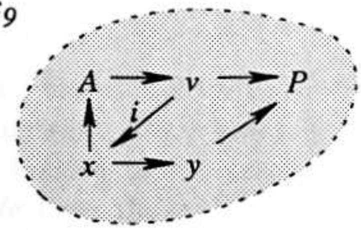


Figure 8: Causal Direction of Bernoulli's Equation

Finally, Figure 8 reveals the stable causal ordering of Bernoulli's equation for liquid flow along a fixed path: $P + \frac{1}{2}\rho v + \rho g y = c_{10}$, where P is pressure, ρ is fluid density, v is velocity, g is the gravitational acceleration constant, and y is height. The variables in this equation represent properties of the fluid at arbitrary points along the path. This equation is based on a highly constrained modeling viewpoint. The liquid flow is idealized to be incompressible [10], steady and nonviscous. The steady flow assumption rules out scenarios where conditions at any location in the path vary over time. For example, the equation cannot model the transient response of a fluid flow system subjected to increased inlet pressure. Bernoulli's equation also appears to be far from structural. It apparently does not represent a distinct physical mechanism through which its variables interact directly. It is not a basic principle, but is derivable from the fundamental laws of Newtonian mechanics.

Nevertheless, a stable causal ordering for Bernoulli's equation exists. The equations of Figure 8 illustrate this. The second equation, $\rho A v = c_{10}$, where A is cross sectional area, is known as the *continuity* equation. It expresses the fact that, at steady flow, the rate of mass flow through any cross sectional area is constant. The third equation, $A = f_1(x)$, where x is position along the path, denotes the fact that the cross sectional area available for flow is a function of position. Similarly, $y = f_2(x)$ says fluid height is also a function of position. In other words, area and height reflect the geometry of the path. Over the scenario space, the position x is influenced by a motion process. That is, x is always quasi-exogenous. Since density ρ is always constant, its causal interactions can be ignored. In contrast, the remaining variables are *endogenous*. That is, they are affected by variables within the mathematical model. External influences on pressure, cross sectional area or height are prohibited because this would violate the steady flow assumption[11]. The equations of Figure 8 form a 5×5 upper triangular structural equation matrix. Thus, by Lemma 1, the equations are unidirectional. In Bernoulli's equation, pressure is causally dependent on velocity and height.

[10]In other words, ρ is constant.

[11]For example, squeezing a garden hose increases its pressure at the point of contact, which causes transient variations in flow.

5 DISCUSSION

The causal ordering procedure of Iwasaki and Simon is a useful tool for uncovering causal dependencies which may be difficult to determine otherwise. Unfortunately, the procedure is context sensitive. That is, the procedure is limited to "...creating models of specific systems in specific modes of behavior"[4]. We have shown that by taking into account underlying modeling viewpoints, it is possible to find clusters of equations whose internal causal orderings are scenario independent. The processes in a domain define quasi-exogenous variables which extend causal ordering's range of validity. Processes impose a direction of causation into the unidirectional equations and dictate which of their variables are gateways for causal propagation. The stable causal ordering within these equations satisfies an intuitive sense that causation should reflect the underlying physics, and should not be due to particular configurations of the outside world. Unidirectional equations help the qualitative modeler by providing independent, unambiguous causal components. These components can be studied in isolation and used as building blocks in many scenarios. To illustrate our ideas, we have uncovered the causal dependencies of Hooke's law, Gauss's law for electricity, and Bernoulli's equation using standard modeling viewpoints of their underlying phenomena.

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15. Supplementary Notes

16. Abstracts

The *causal ordering* procedure of Iwasaki and Simon [5,6] provides a means for uncovering causal dependencies among variables constrained by a set of mathematical equations. This paper examines the procedure from a qualitative modeling viewpoint and addresses one of its limitations: *context sensitivity*. Causal dependencies predicted by the procedure may change depending on the context or scenario in which the underlying physical system operates. This prevents the qualitative modeler from using causal ordering to determine, a priori, a single causal interpretation of equations describing some phenomenon. We show that in some cases it is possible to find clusters of equations that possess *causal stability*. That is, their causal dependencies are the same in all scenarios consistent with the equations' *modeling viewpoint*. These *unidirectional* equations help the qualitative modeler by providing a stable, unambiguous causal interpretation. To identify such equations we define conditions sufficient to guarantee causal stability. In addition, we show that unidirectional equation sets are *causally independent* of equations outside their set. Thus, they add compositionality to the causal modeling task. Lastly, we demonstrate our ideas by uncovering the causal dependencies of Hooke's law, Gauss's law for electricity, and Bernoulli's equation.

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